# Eddy Diffusivity of Solid Particles in a Turbulent Liquid Flow in a Horizontal Pipe

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The transport of particulate matter by suspension in a suitable fluid has been a subject of great interest for many years, both for its many applications as an industrial process and as a natural phenomenon in the erosion of rivers and estuaries (Govier and Aziz, 1982; Raudkivi, 1990; Vanoni, 1975; Wasp et al., 1977).

Many theories and empirical correlations have been developed to account for and to predict parameters of practical and fundamental importance, particularly the frictional pressure drop and the critical velocity below which a bed begins to form (the minimum suspension velocity).

It is generally agreed that at a sufficiently high flow rate, particles remain in suspension because the effect of the turbulent eddies is sufficiently strong to overcome the tendency of the particles to fall under gravity to the bottom of the pipe. Theoretical models of the process describe the process variously in terms of: (1) the turbulent energy required to support the particles; (2) a force on the particles due to the turbulent eddies which balances the force of gravity; and (3) a balance between a downward drift (advection) caused by gravity and diffusion by turbulence against the induced particle concentration distribution.

Oroskar and Turian (1980) developed a formulation based on energy considerations for the minimum suspension velocity in which the unknown parameters were derived from comparison with experimental data. Similarly, Davies (1987) developed a formulation for the minimum suspension velocity based on a force balance in which the unknown parameters were derived from comparison with experimental data. The resulting formulae are similar in form involving the physical parameters of the problem raised to individual powers. The advection/diffusion method has been extensively used to predict the variation with depth of the particle concentration distribution. This method is attractive in that the only parameter that is not very well quantified is the coefficient of diffusivity of the particles due to the turbulent eddies.

The objective of this article is to develop a new and concise expression for the particle eddy diffusivity in turbulent pipe flow. The basis of the calculation procedure is that the advection/diffusion method is used to determine the minimum suspension velocity in terms of the particle eddy diffusivity. Inversion of this result then allows the particle eddy viscosity to be determined in terms of existing empirical or quasi-empirical formulae for the minimum suspension velocity. The effectiveness of the method is emphasized by the simplicity of the result and by the conformity with published data.

## Eddy Viscosity and Particle Eddy Diffusivity

The simplest description of momentum transport in turbulent flow is in terms of the eddy viscosity $\epsilon$ defined by

$$\epsilon = \frac{\tau}{\rho |dU/dy|}. \tag{1}$$

Here $\tau$ is the shear stress, $\rho$ is the fluid density, $U$ is the mean velocity, and $y$ measures distance from the wall. The eddy viscosity is not a property of the fluid alone; it also depends upon the fluid velocity, distance from the wall and the diameter or width $D$ of the conduit.

In general, one may write

$$\epsilon = \bar{\epsilon} F(y/D), \tag{2}$$

where $\bar{\epsilon}$ denotes the mean value of $\epsilon$ over the cross section of the conduit, and the function $F$ represents the variation of $\epsilon$ with distance from the wall; $F$ is defined such that its mean value is 1. On dimensional grounds we expect that

$$\bar{\epsilon} = \bar{\epsilon}_0 DU, \tag{3}$$

where the coefficient $\bar{\epsilon}_0$ depends upon the other dimensionless parameters of the problem, which in this case is only the Reynolds Number $Re$. A solution of this form was obtained by Taylor (1954). Assuming that the velocity is given by the universal velocity profile, he showed that

$$\bar{\epsilon} = 0.026 DU^*, \tag{4}$$

where $U^* = \sqrt{\tau_w/\rho}$ is the friction velocity. It is related to the mean fluid velocity $U$ by

$$U^* = U\sqrt{f/2}, \quad (5)$$

where $f$ is the friction factor. For simplicity a Blasius-type formula may be assumed for $f$,

$$f = aRe^{-b}, \quad (6)$$

where, for example, $a = 0.079$ and $b = 0.25$. Then, Eqs. 4, 5 and 6 reduce to

$$\bar{\epsilon} = 0.026\sqrt{a/2}\,Re^{-b/2}DU, \quad (7)$$

which is precisely of the form proposed above.

By analogy with the eddy viscosity, the eddy particle diffusivity $\epsilon_p$ provides the simplest description of particle transport in turbulent flows. Like the eddy viscosity, the eddy particle diffusivity is not a property of the fluid alone, but depends upon the fluid velocity, distance from the wall and the diameter or width $D$ of the conduit. Also, it is likely that only eddies of a certain scale are effective in transporting particles, and that scale is dependent upon the diameter of the particle $d_p$.

Thus, one may write

$$\epsilon_p = \bar{\epsilon}_p F_p(y/D), \quad (8)$$

where $\bar{\epsilon}_p$ denotes the mean value of $\epsilon_p$ over the cross section of the conduit and the function $F_p$ represents the variation of $\epsilon_p$ with distance from the wall; $F_p$ is defined such that its mean value is 1. On dimensional grounds we expect that

$$\bar{\epsilon}_p = \bar{\epsilon}_{p0} DU, \quad (9)$$

where the coefficient $\bar{\epsilon}_{p0}$ depends upon the other dimensionless parameters of the problem, which in this case are the Reynolds number $Re$ and the ratio of the particle diameter $d_p$ to the conduit diameter $D$. In particular, we shall seek solutions in the form

$$\bar{\epsilon}_{p0} = \bar{\epsilon}_{p00}\left(\frac{d_p}{D}\right)^{\alpha} Re^{\gamma}, \quad (10)$$

where $\bar{\epsilon}_{p00}$, $\alpha$ and $\gamma$ are constants to be determined.

A method of estimating the values of these constants will be developed in the next section and results will be obtained in the following section.

## Particle Suspension by Turbulent Diffusion in Pipe Flow

The model of particle diffusion described above will now be applied to the transport of particles in suspension in a turbulent pipe flow. In particular, we shall be concerned with the critical value of the mean fluid velocity $U_c$ at which bed formation is initiated.

It will be assumed that the mechanism which maintains particles in suspension against the force of gravity is diffusion by turbulent eddies against a vertical particle concentration gradient. In the steady state, turbulent diffusion and the downward movement of the particles due to gravity are in balance leading to the following advection/diffusion equation for the particle concentration $C(y)$:

$$-v_p C = \epsilon_p \frac{dC}{dy}, \quad (11)$$

where $y$ is a coordinate perpendicular to the bed, $v_p$ is the particle settling velocity and $\epsilon_p$ is the particle diffusion coefficient. This model of particle suspension may be traced back to the work of Schmidt (1925), through Rouse (1937), Ismail (1951), and Carstens (1969), to the lucid accounts given in many modern textbooks (see, for example, Wasp et al. (1977), Raudkivi (1990) and Govier and Aziz (1982)).

Generally, the particle settling velocity $v_p$ depends upon the concentration of particles. One of the many representations of the "hindered settling" effect is

$$v_p = v_{p0}(1 - C)^n, \quad (12)$$

due to Richardson and Zaki (1954). Here, $v_{p0}$ is the settling velocity of a single particle, $C$ is the particle concentration and $n$ varies according to the particle Reynolds number.

Equation 11 can be integrated for the concentration $C$ provided that the particle eddy diffusivity $\epsilon_p$ is specified. If $\epsilon_p$ takes the form of Eq. 8, then Eq. 11 may be integrated to give

$$C = C(y/D, Re^*), \quad (13)$$

where

$$Re^* = \frac{v_{p0} D}{\bar{\epsilon}_p}. \quad (14)$$

The precise form of the variation of $C$ with $y/D$ depends upon the form of the hindered settling formula and the function $F_p$ for the distribution of the eddy diffusivity. The constant of integration is determined by applying a suitable boundary condition.

Now suppose that a critical mean flow velocity $U_c$ corresponds to incipient bed formation at the lowest point in the pipe. It will be assumed that at this point the concentration of particles in suspension is equal to the concentration $C_B$ that would be apparent in a bed of particles,

$$C(y/D, Re^*) = C_B \quad \text{at} \quad y = 0. \quad (15)$$

$C_B$ is assumed to be known and equal to the concentration corresponding to cubic packing of identical spheres; a value of 0.52 is used in the calculations. At the point of bed formation all particles are still in suspension. If the mean particle concentration $C_m$ is specified, then $C_m$ is equal to the average of $C(y/d)$ over the cross-sectional area of the pipe. The result is

$$\frac{2}{\pi}\int_{-\pi/2}^{\pi/2} C(y/D, Re^*)\cos^2\theta\, d\theta = C_m \quad \text{at} \quad U = U_c, \quad (16)$$

Table 1. Values of $C_m$ for Various Values of $Re_c^*$

| $C_m$ | $Re_c^*$ | $C_m$ | $Re_c^*$ |
|---|---|---|---|
| 0.0341 | 10.0000 | 0.0859 | 5.0000 |
| 0.0395 | 9.0000 | 0.1119 | 4.0000 |
| 0.0465 | 8.000 | 0.1519 | 3.0000 |
| 0.0557 | 7.0000 | 0.2162 | 2.0000 |
| 0.0682 | 6.0000 | 0.3254 | 1.0000 |

where $y/D = 0.5(1+\sin\theta)$ and $\theta$ is a general angle to the horizontal.

It is apparent that the onset of bed formation occurs at that critical value of $Re^*$ for which Eq. 16 is satisfied from which it follows that

$$Re^* = Re_c^*(C_m) \quad \text{at} \quad U = U_c. \tag{17}$$

The dependence of $Re_c^*$ on $C_m$ is easily found by numerical integration once a functional form has been specified for $F_p(y/d)$. The simplest case is to assume that $F_p = 1$, which corresponds to a uniform particle eddy diffusivity and to ignore hindered settling by setting $n = 0$. The results are shown in Table 1.

If the physical properties of the particle and the fluid are specified, Eqs. 14 and 17 determine the onset of bed formation at a critical value of the mean particle eddy diffusivity $\bar{\epsilon}_{pc}$ where

$$\bar{\epsilon}_{pc} = \frac{Dv_{p0}}{Re_c^*}. \tag{18}$$

The results shown in Table 1 indicate, as expected, that a higher value of the eddy diffusivity is needed to support a higher concentration of particles. The assumption of a different form for $F_p(y/D)$ will lead to a different relation between $Re^*$ and $C_m$, but the functional form of Eq. 18 will be unchanged.

The critical velocity $U_c$ at which bed formation begins is obtained by setting $\bar{\epsilon}_p = \bar{\epsilon}_{pc}$ at $U = U_c$ in Eq. 10. Thus,

$$\bar{\epsilon}_{p00} DU_c \left(\frac{d_p}{D}\right)^{\alpha} Re_c^{\gamma}, = \frac{Dv_{p0}}{Re_c^*} \tag{19}$$

where $Re_c = DU_c/\nu$. On simplification, this becomes

$$\frac{U_c}{v_{p0}} = \frac{1}{Re_c^* \bar{\epsilon}_{p00}} \left(\frac{D}{d_p}\right)^{\alpha} Re_c^{-\gamma}. \tag{20}$$

## Comparison with Experiment and Determination of Parameters

The three parameters $\bar{\epsilon}_{p00}$, $\alpha$ and $\gamma$ which appear in Eq. 10 are as yet undetermined. The basis of the method proposed in this article is that they can be inferred by comparing Eq. 20 with the corresponding equation generated from correlations of empirical data for the onset of bed formation. There are many such formulae in the literature and only the most widely cited and recognized will be considered here.

First, it is useful to recall the standard formulae for the settling velocity of single particles in Newtonian fluids. The terminal settling velocity is given by

$$v_{p0}^2 = \frac{4g(s-1)d_p}{3C_D}, \tag{21}$$

where $s = \rho_p/\rho$ is the ratio of the density of the particle to that of the fluid and $C_D$ is the drag coefficient. A standard approximation to the drag coefficient is

$$C_D = \frac{24}{Re_p} \qquad \text{for } Re_p < 1.4 \tag{22}$$

$$= \frac{21.4}{Re_p^{0.625}} \qquad \text{for } 1.4 < Re_p < 500 \tag{23}$$

$$= 0.44 \qquad \text{for } Re_p > 500, \tag{24}$$

from which it follows that

$$v_{p0} = g(s-1)d_p^2/18\nu \qquad \text{for } Re_p < 1.4 \tag{25}$$

$$= 0.13[g(s-1)]^{0.72} d_p^{1.18} \nu^{-0.45} \quad \text{for } 1.4 < Re_p < 500 \tag{26}$$

$$= 1.74[g(s-1)d_p]^{0.5} \qquad \text{for } Re_p > 500. \tag{27}$$

The particle Reynolds Number is defined by $Re_p = v_{p0}d_p/\nu$. Stokes' Law (Eq. 25) is strictly valid for $Re_p \ll 1$, but it is sufficiently accurate for our purposes up to $Re_p = 1.4$ at which point the predicted settling velocity is equal to that given by the empirical formula for intermediate values of the particle Reynolds Number (Eq. 26). Also, Newton's formula (Eq. 27) is usually applied to $Re_p > 1{,}000$, but here is used for $Re_p > 500$ at which point the predicted settling velocity is equal to that given by the empirical formula for intermediate values of the particle Reynolds Number.

For sand particles in water (for which $s = 2.65$), we note that Stokes' Law applies for particles of diameter less than 0.18 mm and Newton's formula may be used for particles of diameter larger than 1.7 mm.

Oroskar and Turian (1980) developed a correlation for $U_c$ from an analysis of 357 published experimental data points. The data encompassed several different fluids and particles, but the particle settling velocity for the large majority of the data fell into the intermediate category given by Eq. 26 above. Their correlation for $U_c$ may be written as

$$U_c \propto [g(s-1)]^{0.55} d_p^{0.17} D^{0.47} \nu^{-0.09}. \tag{28}$$

In some respects this can be regarded as an extension of earlier and much quoted correlations due to Durand (1952) and Wicks (1968) (see, for example, Wasp, 1977, p. 89). Durand's correlation is

$$U_c \propto [g(s-1)]^{0.5} D^{0.5} \tag{29}$$

and Wicks' correlation is

$$U_c \propto [g(s-1)]^{0.5} D^{0.5} (d_p/D)^{1/6}. \quad (30)$$

The constant of proportionality in Durand's correlation increases slowly with $d_p$, which is consistent with Wicks' correlation. Both correlations are similar to Oroskar and Turian's correlation, except that the older formulae contain no explicit dependence on fluid viscosity.

Oroskar and Turian's correlation can be translated into the form of Eq. 20 by employing Eq. 26 for the particle settling velocity. From Eq. 26,

$$[g(s-1)]^{0.72} = 5 v_{p0} d_p^{-1.18} \nu^{0.45}. \quad (31)$$

Substitution into Eq. 28 gives

$$U_c^{1.32} \propto v_{p0} d_p^{-0.96} D^{0.62} \nu^{0.33} \quad (32)$$

With only a very small adjustment to the empirical coefficients (1.32 to 1.33 and 0.62 to 0.63), this result can be written in the form of Eq. 20

$$\frac{U_c}{v_{p0}} = \text{constant} \left( \frac{D}{d_p} \right)^{\alpha} Re_c^{-\gamma}. \quad (33)$$

in which

$$\alpha = 0.96 \quad \gamma = 0.33. \quad (34)$$

Oroskar and Turian also developed a formula for $U_c$ similar in form to their correlation (Eq. 28), but ultimately differing in the estimate of the exponents. It is equivalent to

$$U_c \propto [g(s-1)]^{0.53} D^{0.6} \nu^{-0.07}. \quad (35)$$

The theoretical development behind the formula is based on a balance between the energy required to maintain the particles in suspension and the turbulent energy expended by the flow. Again, by assuming that the majority of the data falls into the intermediate regime, the result can be cast into the form of Eq. 20 with, now

$$\alpha = 1.18 \quad \gamma = 0.35. \quad (36)$$

Davis (1987) developed a theory for the critical suspension velocity based on a balance between the sedimentation force on a particle and an "eddy fluctuation force." It is implicitly assumed that the particles are stationary, and no account is taken of hydrodynamic drag on the particles. The effective eddy length is assumed to be equal to the particle diameter. Davies' formula

$$U_c \propto [g(s-1)]^{0.54} d_p^{0.18} D^{0.46} \nu^{-0.09}. \quad (37)$$

turned out to be very similar to Oroskar and Turian's correlation (Eq. 28).

Earlier, Thomas had explored the transport properties of suspensions in a long series of articles. In Thomas (1962) he gave the following correlation for the critical transport velocity for large particles (0.2 mm to 38 mm) in water at infinite dilution:

$$\frac{U_c}{v_{p0}} \propto (s-1)^{-0.23} \left( \frac{D}{d_p} \right) Re_c^{-0.4} \left( \frac{U_c}{U_c^*} \right)^{0.6}. \quad (38)$$

A correction term was applied to allow for the effect of finite particle concentration. The friction velocity $U^*$ is related to the mean velocity $U$ by Eqs. 5 and 6.

It is not clear how the term in $(s-1)$ should be treated in the present context. It could be eliminated by using Eq. 26 for the particle settling velocity, but the result would not be dimensionally correct without the introduction of $g$, which does not enter our final formulation in Eq. 20. It is perhaps safer to assume that the correlation applies only to a specific value of $s$ (for sand particles in water, for example) and to include this term in the constant of proportionality. On this basis, Eq. 38 may be written in the form

$$\frac{U_c}{v_p} = \text{constant} \left( \frac{D}{d_p} \right) Re_c^{0.3b-0.4}. \quad (39)$$

This is consistent with Eq. 20 if

$$\alpha = 1.0 \quad \gamma = 0.4 - 0.3b \quad (40)$$

In the Blasius formula for the friction factor, $b$ takes the value of 0.25, which gives $\gamma = 0.33$.

These empirically-based predictions of $\alpha$ and $\gamma$ are summarized in Table 2.

There is clearly good agreement between the empirical correlations of Thomas and Oroskar and Turian and the predictions of their own and Davies' theoretical models. On the basis of these results then, the following values are proposed

$$\alpha = 1.0 \quad \gamma = 1/3. \quad (41)$$

The value of the remaining parameter $\bar{\epsilon}_{p00}$ is obtained by making a comparison of the predictions of the model with available data. Thus $U_c$ has been calculated from Eqs. 16 and 20 and the value of $\bar{\epsilon}_{p00}$ varied to obtain the best agreement with the data. The data selected for comparison is taken from Davies (1987) and covers a wide range of particle and pipe diameters for sand suspensions in water at a volumetric concentration of 12%. The best value of $\epsilon_{p00}$ was found to be 0.014. Table 3 compares the predictions from the theory with the experimental data and Davies' formula. The experimental data represent means of the data sets presented by Davies in

**Table 2. Values of $\alpha$ and $\gamma$ on the Basis of Various Empirical Data Sets or Theory**

| Source | $\alpha$ | $\gamma$ |
|---|---|---|
| Oroskar and Turian (1980) (expt) | 0.96 | 0.33 |
| Oroskar and Turian (1980) (theory) | 1.18 | 0.35 |
| Davies (1987) (theory) | 0.96 | 0.33 |
| Thomas (1962) (expt) | 1.0 | 0.33 |

**Table 3. Suspension Velocity in Horizontal Pipe***

| $d_{sand}(\mu)$ | $d_{pipe}$(mm) | $v_{exp}$(m/s) | Davies | Walton |
|---|---|---|---|---|
| 100 | 26 | 0.8 | 1.0 | 0.7 |
| 130 | 19 | 0.7 | 0.9 | 0.8 |
| 130 | 105 | 1.5 | 1.9 | 1.7 |
| 200 | 202 | 2.2 | 2.8 | 2.4 |
| 220 | 105 | 1.7 | 2.1 | 1.8 |
| 230 | 26 | 0.9 | 1.1 | 1.0 |
| 250 | 410 | 3.2 | 4.0 | 3.3 |
| 280 | 700 | 4.3 | 5.3 | 4.3 |
| 400 | 103 | 1.8 | 2.3 | 1.9 |
| 420 | 308 | 3.1 | 4.0 | 3.1 |
| 420 | 206 | 2.6 | 3.3 | 2.6 |
| 440 | 150 | 2.2 | 2.9 | 2.2 |
| 570 | 51 | 1.4 | 1.8 | 1.4 |
| 590 | 108 | 2.0 | 2.6 | 2.0 |
| 650 | 26 | 1.0 | 1.4 | 1.1 |
| 750 | 89 | 1.9 | 2.4 | 1.9 |
| 850 | 76 | 1.8 | 2.3 | 1.8 |
| 1,200 | 19 | 1.0 | 1.3 | 1.0 |
| 1,200 | 108 | 2.2 | 2.9 | 2.2 |
| 1,350 | 407 | 4.3 | 5.4 | 4.0 |
| 2,000 | 150 | 2.8 | 3.8 | 2.5 |

*Comparison of experimental data from Oroskar and Turian (1980) with Davies' (1987) theoretical result and results of present calculation.

his Table 1. The agreement between the present theory and the experimental data is very good and is in fact slightly better than between Davies' theory and experiment.

The final result for the minimum suspension velocity is

$$\frac{U_c}{v_{p0}} = \frac{71.2}{Re_c^*}\left(\frac{D}{d_p}\right)Re_c^{-1/3}. \tag{42}$$

## Discussion

By using a model of particle suspension in which advection of the particles by gravity is balanced by diffusion due to turbulent eddies, a new estimate has been developed for the coefficient of diffusivity of the particles in a turbulent pipe flow. The resulting formula is

$$\bar{\epsilon}_p = \bar{\epsilon}_{p0} DU, \tag{43}$$

where

$$\bar{\epsilon}_{p0} = \bar{\epsilon}_{p00}\left(\frac{d_p}{D}\right)Re^{1/3}. \tag{44}$$

Some authors in the field of sediment transport simply equate the eddy particle diffusivity $\epsilon_p$ to the eddy momentum diffusivity (eddy viscosity) $\epsilon$. Others treat them as proportional by writing

$$\epsilon_p = \beta\epsilon, \tag{45}$$

where $\beta$ is a constant of proportionality (see Wasp, 1977; Raudkivi, 1990; Govier and Aziz, 1982). There is a suggestion in Wasp that $\beta$ increases with particle size, and this might be expected on the basis that larger particles will only be affected by the large-scale eddies which will tend to distribute them quickly and efficiently over the whole cross section. A more precise estimate of $\beta$ can be obtained from the present theory. Suppose that the distribution of the eddy viscosity and the eddy diffusivity across the pipe are identical. Then Eq. 45 gives

$$\beta = \frac{\bar{\epsilon}_p}{\bar{\epsilon}}. \tag{46}$$

According to Taylor (1954) $\bar{\epsilon}$ takes the form

$$\bar{\epsilon} = 0.026DU^*. \tag{47}$$

Using the Blasius formula for the friction factor, this becomes

$$\bar{\epsilon} = 0.026\sqrt{a/2}\,DURe^{-b/2}. \tag{48}$$

The average eddy diffusivity is given by Eqs. 43 and 44. Consequently, $\beta$ is given by

$$\beta = \frac{\bar{\epsilon}_{p00}}{0.026\sqrt{a/2}}\left(\frac{d_p}{D}\right)Re^{b/2+1/3}. \tag{49}$$

This result confirms that $\beta$ increases with the particle diameter.

In the course of the development of the theory leading to Eqs. 43 and 44, it was necessary to make some assumptions about the spatial distribution of the eddy diffusivity and about the form of the hindered settling formula. Neither of these affects the form of the final solution, but it is expected that they do modify the value of the coefficient $\epsilon_{p00}$. Also, $\epsilon_{p00}$ has been evaluated by comparing with data at a single particle concentration; in general, it is expected that $\epsilon_{p00}$ varies with the particle concentration.

Finally, the minimum suspension velocity is given by

$$\frac{U_c}{v_{p0}} = \frac{71.2}{Re_c^*}\left(\frac{D}{d_p}\right)Re_c^{-1/3}. \tag{50}$$

It should be noted that this apparently simple expression is in fact implicit in $U_c$, but can be easily rearranged to give

$$\frac{U_c^{4/3}}{v_{p0}} = \frac{71.2}{Re_c^*}\left(\frac{D}{d_p}\right)\left(\frac{\nu}{D}\right)^{1/3}. \tag{51}$$

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## Notation

$a$ = coefficient in Eq. 6
$b$ = coefficient in Eq. 6
$D$ = pipe diameter
$f$ = friction factor

$g$ = acceleration due to gravity
$n$ = coefficient in Eq. 12
$y$ = distance from wall
$\alpha$ = coefficient in Eq. 10
$\beta$ = particle diffusivity/eddy viscosity
$\gamma$ = coefficient in Eq. 10
$\theta$ = angle to the horizontal
$\nu$ = kinematic viscosity
$\rho$ = fluid density
$\tau$ = shear stress

### *Subscripts*

$c$ = critical (cf. bed formation)
$w$ = wall

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